Public Transportation Advertisement Scheduling: Algorithms and a Case Study in Taiwan

Completed Research Paper

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Abstract

We consider a real-world advertisement scheduling problem on public transportation. With modern technology of face recognition, the number of effective exposures of each advertisement display may be estimated more accurately than before. The objective of this work is thus to maximize the minimum effective exposure among advertisers to preserve fairness, one common operating issue faced by media agencies. We propose an algorithm consisting of two parts and aim to handle both efficiency and efficacy. The first part is a heuristic modified from the famous longest processing time (LPT) rule. The second part is a branch-and-bound algorithm by adjusting optimality gap over runtime to obtain the best possible schedule within the time limit. We run numerical experiments to evaluate the performances of the algorithm. Collaboration with a Taiwanese media agency on bus routes demonstrates the applicability of our algorithm in practice.

Keywords: advertisement scheduling, public transportation, passenger composition prediction, integer programming, machine learning

Introduction

Advertising is an approach widely adopted by companies to promote their products and services and boost their public image. Among existing media, including print media, televisions, radio, and the

Internet, televisions provide a relatively vibrant way of advertising and are thus favored by advertisers. Traditionally, one major concern of advertising on televisions is the way of scheduling. A media agency is usually given a set of commercials to display and needs to fit those commercial into time slots between television programs using some scheduling methods. Its schedule also has to meet the needs of its clients.

In recent years, the advent of the Internet enables advertisers to reach millions of users in a short period of time, which leads online advertising to the most popular advertising medium. Furthermore, with advancement of technology, targeted advertising becomes viable. Many publishers like social media have attempted targeted advertising based on user behavior data. Hence, the main focus of online advertising is the use of user data. When users sign up on websites, they are asked whether they agree to provide personal data for commercial use, with which these publishers can improve advertising.

Public transportation is also an increasingly used medium for advertising. While online advertising can reach millions of users and television commercials provide vivid contents, public transportation shares both benefits. Recently, some media agencies have considered advertising on buses. According to Media Agencies Association of Taipei (MAA), the Internet is the leading medium in 2017, followed by televisions and outdoors. Moreover, the average number of people commuting with buses in Taipei, Taiwan is 1,302,832 per day, which is about half of Taipei city population. The average number of daily bus commuter further hikes to over 8.37 million in Beijing, China. There are 2.23 billion passenger journeys in 2017 in London, UK. In Seoul, Korea, from January to October in 2017, there are more than 150 million bus commuters in each month.[1]All these statistics indicates the importance of buses to commuters and the business potential lies in advertising on bus services.

When advertisers sign contracts with media agencies, they usually state their demand such as commercials should be displayed within a certain time period or receive a certain amount of media exposure. Media agencies will then insert commercials into available time slots. In the past, because there is no direct way for a media agency to understand the audience in from of the screens, most agencies tend to adopt cyclic broadcasting. Its inefficiency is obvious, especially for public transportation like buses. After all, when a bus is travelling in a city, the passenger composition may differ significantly at different locations. Different advertisements should be displayed when a bus is full of young students heading for schools or old persons just leaving hospitals. With modern technology of face recognition, nowadays media agencies are able to retrieve passengers' demographic profiles (age and gender) to identify advertisers' target customers and then display corresponding commercials. Advertisement scheduling on public transportation therefore faces new challenges and chances.

In this research, we consider real-world bus service problems with passenger demographic information. While the number of effective exposures may be better estimated when each advertisement is displayed, a media agency adds this information into its advertisement scheduling problem. The most important feature of our advertisement scheduling problem is that the decision maker considers not only the overall profitability but also fairness among advertisers. Bollapraga and Garbiras (2004) have tackled the fairness issue in advertisement scheduling on television. They state that, in a single commercial break, the first and last segments usually reach more viewers and are thus more valuable to advertisers. Since it is hard to identify whether viewers are target customers, they propose to assign these segments to all commercials evenly whenever possible. Our conversation with a Taiwanese media agency also indicates the same concern (see the section of Case Study at the end of this study).

For our problem, we aim to display most suitable commercial based on the passenger composition. We choose effective exposure as the indicator to evaluate how much the commercial is exposed to its target customers. Effective exposure is defined as the summation over the product of the number of each type of passengers and passenger preference toward the commercial. Our objective is to maximize the

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[1] The information about MAA is from *https://maataipei.org/*. The information about bus statistics in Taipei, Taiwan is from *https://english.dot.gov.taipei/*. The information about bus statistics in Beijing, China is from *https://www.bjbus.com/*. The information about bus statistics in London, UK is from *https://www.gov.uk/government/collections/bus-statistics*. The information about bus statistics in Seoul, Korea is from *http://data.seoul.go.kr/*

minimum effective exposure among all commercials to preserve fairness among advertisers as much as possible. We design a heuristic to solve our advertisement scheduling problem. The heuristic consists of two parts. The first part is a greedy algorithm based on the LPT rule to provide an initial solution to our problem. The second part is a branch-and-bound algorithm by adjusting optimality gap over runtime to further improve the schedule. To evaluate the proposed algorithm, we conduct a numerical experiment and implement our algorithm in collaboration with a media agency. For the numerical experiment, we consider 324 scenarios generated by varying five factors, and test well against three benchmark algorithms. For the application, we work with the media agency, and implement our algorithm to produce schedules for two bus routes. Hence, our proposed algorithm has demonstrated its applicability in practice and capability of handling both efficacy and efficiency.

In the next section, we will review some related works. In Problem Description, we formulate our advertisement scheduling problem as an integer program. We propose two algorithms for solving our problem in Algorithms. We then conduct a numerical experiment to demonstrate the performance and general applicability of our proposed algorithm in Numerical Experiments. Case Study describes our collaboration with a Taiwanese media agency and implementation of our algorithm. Finally, we conclude this study and address some possible extensions.

Literature Review

Television advertisement scheduling is similar to our advertisement scheduling problem, in which we have to insert commercials to available time slots given some specific goals. Advertisement scheduling on televisions has been widely studied in literature. Typically, there are several points we focus on during the scheduling process. Reddy *et al.* (1998) propose a general model where we can set our objective based on factors such as revenue or customer reaction. The maximization of revenue is also discussed by Alaei and Ghassemi-Tari (2011). They look upon this problem as a combination auction problem and use genetic algorithm to test its efficacy. Another factor of advertisement scheduling is the interval between same commercials. Bollapraga *et al.* (2004) focus on inserting commercials evenly, which makes the intervals between same commercials as large as possible. In addition, advertisers' demands also play an important role in advertising scheduling. For example, an advertiser may not want to display its commercial next to its opponents' that promote similar products. This factor is known as advertising conflict. Advertising conflict is discussed by Gaur *et al.* (2009). They provide a scheduling heuristic, dealing with the conflicts between all pairs of commercials. Other than conflicts, a customer's goal is also used to propose a model. Brusco (2008) considers the failure to meet customer goal and advertising conflicts as penalties and use a branch-and-bound algorithm to minimize them.

Online advertisement scheduling also resembles our problem and is widely discussed in literature. Some uses customer data to formulate the model. Different from problems of advertising on televisions, most online advertisement scheduling researches schedule banners instead of video commercials. Adler *et al.* (2002) propose a problem to schedule banners given their size and display frequency. Amiri *et al.* (2003) perform Lagrangian decomposition to improve the original branch-and-bound method. Dawande *et al.* (2003) then give two possible goals of this problem. One is MINSPACE problem, which aims to minimize the space it takes to fit all commercial in slots with 2-approximation algorithm, which is then improved to $1+1\sqrt{2}$ - approximation (Dawande *et al.*, 2005). The other is MAXSPACE problem, whose goal is to maximize the space given a maximal space banners can occupy. Some researchers also use user data to perform scheduling. Kazienko *et al.* (2007) utilize user history to find possible commercial that a user may be interested in. Turner (2012) also aims to meet certain extent of exposure of different customer groups.

We consider scheduling multiple commercials in several available time slots. For each available time slot, we choose the most suitable commercial to display based on expected effective exposure it receives. As the objective is to maximize the minimum effective exposure, we treat this problem as a variation of the machine scheduling problem. Assume each commercial is equivalent to a machine and each available time slot is a job. Assigning a time slot to a commercial becomes assigning a job to a machine. Effective exposure the chosen commercial receives is similar to processing time a machine needs. A special case of our problem considers number of passengers exposed as the sole measurement of effective exposure. In this case, inserting different commercials into the same time slot does not alter effective exposure. Hence, the problem reduces to maximizing the minimum completion time on parallel identical machines, which is the $P||C_{min}$ problem solved by some researchers using the branch-and-bound algorithms. $P||C_{min}$ has been first described by Deuermeyer *et al*. (1982) and they show that the worst solution of the algorithm to never less be ¾ times the optimal one. Csirik *et al.* (1992) prove that the minimum completion time of the LPT-schedule is at least (3*m*-1)/(4*m*-2) times the optimal minimum completion time where *m* denotes the number of machines. Haouari and Jemmali (2008) set tight upper and lower bound and develop a symmetry-breaking branching strategy. Walter (2013) proposes a restricted LPT (RLPT) and proves that it has the approximation factor 1/*m*. Walter *et al.* (2017) further improve the performance by enhancing upper and lower bounds and implementing a depth-first branching scheme.

One of the differences between our problem and the above studies is that our problem considers utilizing user data to perform targeted advertising. Secondly, unlike online advertisement scheduling focusing on space usage and assignment, we put emphasis on avoiding excessive advertisement repetition and product conflicts. Another main characteristic of our problem is that passengers have varying preferences toward different commercials. Due to passenger composition and passenger preferences, inserting different commercials into the same time slot may result in different effective exposure, similar to assigning a job to non-identical machines requires different processing time. Therefore, we include the longest processing time (LPT) rule in our algorithm.

Problem Description

Formulation

We consider a problem of inserting M commercials into N time slots, with T being the duration, which is predefined by most media agencies to set prices, of a single time slot. However, in the real world, we seldom get the number of time slots directly; instead we get D_l ,which is defined as the time it takes to travel from the first stop to the l^{th} stop. We can actually obtain N by some calculation with T and D_l ($l \in \{1,...,L\}$). For example, if we know the duration of a single travel is an hour, D_L = 3600 (seconds), and T is 10 seconds, N thus will be 360, which is 3600 divided by 10.

Supposed all passengers can be clustered into K passenger types, we define $A_{lk}(l \in \{1,...,L\}, k \in \{1,...,K\})$ as the number of passengers from customer type k between l^{th} and $l+1^{th}$ stop. We let W_j ($j \in \{1,...,M\}$) be the time slots it take to display commercial *j*, and let P_{jk} be preference of customer type k toward commercial *j*. In the following formulation, we use $I = \{1,...,N\}$ to be the set of all time slots, and $J = \{1,...,M\}$ to be the set of all commercials.

To deal with time real-world limitations stated later, we first need to define two binary variables.

x_{ij}: $1\ if\ commercial\ j\ is\ played\ in\ time\ slot\ i; 0, otherwise, \qquad \forall i \in I, \forall j \in J$

y_{ij}: $1\ if\ commercial\ j\ starts\ to\ be\ played\ in\ time\ slot\ i; 0, otherwise, \qquad \forall i \in I, \forall j \in J$

We then define effective exposure, which is strongly related to our objective function. When a commercial j is played one time, we let its effective exposure be the product between numbers of all passenger types and each preference toward commercial *j*. Therefore, effective exposure commercial j receives in one travel is

$$\sum_{l \in L} \sum_{i=D_{l-1}+1}^{D_l} y_{ij} \sum_{k \in K} A_{lk} P_{jk} \qquad \forall j \in J.$$

For M commercials, each has its own effective exposure. That the minimal effective exposure be z, the objective is to maximize z, which preserves allocation fairness. Below we consider several constraints.

Excessive Advertisement Repetition

As consumers tend to have a negative impression of a commercial displayed multiple times in a row, that each commercial is allowed to be displayed up to B times successively, i.e.

$$\sum_{i_2=1}^{B+1} y_{(i+i_2\times W_j)j} \leq B \qquad \forall i, 1 \leq i \leq N - BW_j, \forall j \in J.$$

Moreover, a commercial being displayed frequently within certain time span also generates negative effects to consumers. Hence, that Q be a given time span, and R be a given frequency limit, we have the constraint:

$$\sum_{i}^{i+Q} y_{ij} \leq R \qquad \forall j \in J.$$

Product Conflicts

Some commercials feature similar products and one of them is scheduled to be displayed after the other one, product conflicts may emerge. To avoid this situation, we impose the following constraint. Suppose commercials j_1 and j_2 have conflicts with each other, and F be the number of time slots they need to be away from each other. That parameter $G_{j_1j_2}$ as 0 when the conflict between commercials j_1 and j_2 exists and 1 otherwise. Furthermore, the third binary variable is needed to determine which commercial is not displayed during F time slots if confliction exists ($G_{j_1j_2} = 0$).

$$c_{ij_1j_2}\colon \begin{cases} 1, \text{from time slots } i \text{ to } i+F, \text{commercial } j_2 \text{ cannot be displayed} \\ 0, \text{from time slots } i \text{ to } i+F, \text{commercial } j_1 \text{ cannot be displayed} \end{cases} \quad \forall i \in I, \forall j_1, j_2 \in J.$$

Therefore, we have two constraints to handle the conflict between commercials j_1 and j_2:

$$\sum_{i_2=0}^{F-1} y_{(i_1+i_2)j_1} \leq N\left(G_{j_1j_2} + c_{i_1j_1j_2}\right) \qquad \forall i_1 \in I, \forall j_1, j_2 \in J.$$

$$\sum_{i_2=0}^{F-1} y_{(i_1+i_2)j_2} \leq N\left(G_{j_1j_2} - c_{i_1j_1j_2} + 1\right) \qquad \forall i_1 \in I, \forall j_1, j_2 \in J.$$

Model

Collectively, our advertisement scheduling problem can be formulated as

$$\begin{aligned}
\max\ & z \\
\text{s.t.}\ & \sum_{i_2=1}^{B+1} y_{(i+i_2\times W_j)j} \leq B && \forall i, 1 \leq i \leq N - BW_j, \forall j \in J && (1) \\
& \sum_{i}^{i+Q} y_{ij} \leq R && \forall j \in J && (2) \\
& \sum_{i_2=0}^{F-1} y_{(i_1+i_2)j_1} \leq N(G_{j_1j_2} + c_{i_1j_1j_2}) && \forall i_1 \in I, \forall j_1, j_2 \in J && (3) \\
& \sum_{i_2=0}^{F-1} y_{(i_1+i_2)j_2} \leq N\left(G_{j_1j_2} - c_{i_1j_1j_2} + 1\right) && \forall i_1 \in I, \forall j_1, j_2 \in J && (4) \\
& \sum_{j \in J} x_{ij} \leq 1 && \forall i \in I && (5) \\
& y_{ij} \leq x_{ij} && \forall i \in I, \forall j \in J && (6) \\
& y_{i_1j} \leq x_{(i_1+i_2)j} && \forall i_1 \in I, \forall i_2 \in \{0, \ldots, W_j - 1\}, \forall j \in J && (7) \\
& y_{i_1j} + y_{(i_1+i_2)j} \leq 1 && \forall i_1 \in I, \forall i_2 \in \{0, \ldots, W_j - 1\}, \forall j \in J && (8)
\end{aligned}$$

$$x_{i_1 j} \leq \sum_{i_2=0}^{W_j-1} y_{(i_1-i_2)j} \qquad \forall i_1 \in I, \forall j \in J \tag{9}$$

$$z \leq \sum_{l \in L} \sum_{i=D_{l-1}+1}^{D_l} y_{ij} \sum_{k \in K} A_{lk} P_{jk} \qquad \forall j \in J \tag{10}$$

$$x_{ij} \in \{0,1\} \qquad \forall i \in I, \forall j \in J \tag{11}$$

$$y_{ij} \in \{0,1\} \qquad \forall i \in I, \forall j \in J \tag{12}$$

$$c_{ij_1j_2} \in \{0,1\} \qquad \forall i \in I, \forall j_1, j_2 \in J \tag{13}$$

The first four constraints correspond to the real-world limitations discussed above. The fifth constraint defines x_{ij} and ensures only one commercial displayed in each time slot. From the sixth to ninth constraints are definitions of y_{ij}. The sixth constraint states that if commercial j starts to be broadcast in time slot i ($y_{ij} = 1$), then it is being broadcast in the same time slot ($x_{ij} = 1$). The seventh and eighth constraints ensure that if commercial j starts to be displayed in time slot i_1, we cannot halt the broadcast. The ninth constraint makes sure that commercial j starts to be broadcast in time slot i_1, x_{ij} is set as 1 during the broadcast. As our objective is to maximize the minimum effective exposure, the tenth constraint designates z be the minimum effective exposure.

Algorithms

As our advertisement scheduling problem is a strongly NP-hard problem, we design a heuristic capable of handling both efficacy an efficiency. This heuristic consists two parts. The first part is a greedy algorithm based on the classic LPT rule which provides an initial solution to our problem in a few seconds. The second part puts emphasis on efficacy and uses a branch-and-bound algorithm by adjusting optimality gap over runtime and attempt to improve the greedy solution within a given time limit.

For the first part, when deciding which commercial to be broadcast in available time slots, we rank all the commercials by each current effective exposure receives. If several commercials share the same amount of effective exposure, indices decide the priority. In the first half of total time slots, we pick three candidates with lowest effective exposure and choose the one with the highest increase. In the last half, the commercial with the lowest effective exposure will be directly put into schedule. As scheduling finishes, the algorithm makes additional local adjustments to the final schedule. Throughout the schedule, we attempt assigning time slots occupied by other commercials to the one with lowest effective exposure and observe whether the performance is improved. If the re-assignment improves the performance, we then modify the schedule. Since the aim is to improve the minimum effective exposure, this process is repeatedly performed until there is little space for enhancement.

For the second part, we develop a branch-and-bound algorithm to solve the integer program and attempt to improve the greedy solution. Since the integer program is hardly solvable in polynomial time, our methodology is to adjust the optimality gap over runtime. Initially, the optimality gap limit is set to 0.5%, which means the solver terminates when it finds the solution with the gap no more than 0.5%. After 2 minutes, we raise the limit to 1%. As every 1 minute passed, we raise the limit by 1%, up to 5%. In other words, the adjustment stops after the runtime reaches six minutes. The solver terminates when the runtime comes to 8 minutes regardless of optimality gap. We use (2,1,8) to denote the parameter set, where the adjustment starts at the second minute, raises the optimality gap limit by 1% every minute passed, and the time limit is eight minutes. For ease of exposition, we denote this algorithm as the branch-and-bound with self-adjusting optimality gap (BAOG) algorithm.

The second part of BAOG aims to improve the greedy solution. It is possible that the solver is not able to find a feasible solution within the given time limit, if either the number of commercials or the number of time slots is too large. We can tune the parameter set to improve performance of BAOG.

Numerical Experiments

To understand how BAOG performs on the advertisement scheduling problem, we consider the following five factors. We define each factor as follows.

The first factor is the number of time slots, we consider three scenarios, each with 360-720, 720-1080, 1080-1440 time slots. The second factor is the number of commercials, we consider there are 10, 25, or 50 commercials being scheduled. The third factor is the number of passenger types. We categorize passengers into 12 types according to age and gender. We then consider three scenarios where passengers onboard belong to 1 type, one out of 4 or 12 types. The fourth factor focuses on how passenger preferences toward commercials affect the performance. We consider four scenarios, the first one suggests all preferences are randomly distributed between 1 to 5. In the second scenario, each passenger type has extreme preference toward different advertisements. In the third scenario, each passenger type either hates or loves all the advertisements. In the last scenario, each advertisement is either hated or loved by all passenger types. The last factor is the competitive relationship between advertisements. We consider three scenarios, in which there are 0%, 10%, 25% chance that two advertisements have product conflicts.

These five factors together generate 324 scenarios for numerical experiments. We generate 10 random instances for each scenario. We run each instance using BAOG and compare with other three benchmark algorithms. Each instance includes the passenger composition between bus stops, passenger preferences toward commercials, the competitive relationship between commercials, number of time slots, number of stops, number of commercials, and number of passenger types.

We solve the problem with solver Gurobi running on a Core i5-8400 computer with a 2.8GHz processor, 16 GB of RAM and a Windows 10 operating system. We use Python 3.6 to implement BAOG and other three benchmark algorithms and invoke Gurobi.

Benchmark Algorithms

Three benchmark algorithms are used to test against our algorithm BAOG.

Since cyclic broadcasting is commonly used in business operation, we adopt it as the first benchmark algorithm. We assign an index to each advertisement, which forms a fixed display order. Advertisements will be broadcast in order repeatedly until there is no available time slot.

Genetic algorithm is the second benchmark algorithm. We generate a pool of 50 stochastic feasible solutions and randomly pick two out of five best feasible solutions to perform a crossover. In genetic recombination, we randomly designate a time slot as the crossover point. One offspring is bred by appending a parent's slots to the right of that point to the left part of the other parent. Furthermore, the offspring has a 2% chance of becoming a mutant. When the mutation happens, at a certain time slot the advertisement scheduled will be replaced by another one. During each successive generation, five offspring are added to the pool and the population removes five worst solutions. The algorithm terminates when 500 generations are reached or the runtime lasts for six minutes.

For the third benchmark algorithm, we adopt the greedy algorithm introduced in the previous section. Since the greedy algorithm is the first part of BAOG, it is trivial that BAOG certainly outperforms this benchmark algorithm. We include this algorithm to demonstrate the initial performance of BAOG and most importantly, how well the second part of BAOG improves the performance.

Analysis

Throughout this section, we denote z^*as the objective value of the linear relaxation solution, which is solved by Gurobi, and z^{BAOG} as that of BAOG solution, z^{C}, z^{GA}, z^{LPT}as that of cyclic broadcasting solution, genetic solution, and greedy solution respectively.

Number of Time Slots

In any scenario, our algorithm outperforms other three algorithms. For BAOG, when the number of time slots increases, BAOG performs better. This is because as a one-way trip takes less hours, every decision has a greater impact on the performance. By observing the difference of each algorithm under these three factors, we find that BAOG maintains its performance level well.

Table 1. The Effect of Number of Time Slots

Number of Time Slots	Average			
	z^C/z^*	z^{GA}/z^*	z^{LPT}/z^*	z^{BAOG}/z^*
360-720	0.592	0.500	0.875	0.960
720-1080	0.610	0.593	0.926	0.970
1080-1440	0.612	0.630	0.943	0.975

Number of Commercials

In any scenario, our algorithm outperforms other three algorithms. By observing Table 2, we find that all algorithms perform worse as the number of commercials increases. This observation can be explained by a universal phenomenon that if we allocate finite resources to more people, each person is granted less resources. However, the BAOG performance decline is smaller than that of others.

Table 2. The Effect of Number of Commercials

Number of Commercials	Average			
	z^C/z^*	z^{GA}/z^*	z^{LPT}/z^*	z^{BAOG}/z^*
10	0.659	0.809	0.961	0.994
25	0.601	0.598	0.927	0.978
50	0.554	0.316	0.856	0.933

Passenger Types

In any scenario, our algorithm outperforms other three algorithms. Table 3 shows that it is hard for cyclic broadcasting and genetic algorithm to reach target customers when the majority of passengers belong to a certain customer type. On the other hand, the greedy algorithm improves roughly by 3%, and BAOG remains its performance level.

Table 3. The Effect of Number of Passenger Types

Number of Passenger Types	Average			
	z^C/z^*	z^{GA}/z^*	z^{LPT}/z^*	z^{BAOG}/z^*
1	0.542	0.550	0.934	0.973
4	0.591	0.584	0.908	0.968
12	0.682	0.589	0.901	0.964

Passenger Preference Toward Commercials

In any scenario, our algorithm outperforms other three algorithms. From Table 4, we find that while BAOG remains its performance level, other three benchmark algorithms perform their best when each passenger type either hates or loves all the commercials (Situation C). Cyclic broadcasting and genetic algorithm perform their worst when each commercial is either hated or loved by all passenger types (Situation D).

Table 4. The Effect of Passenger Preference

Passenger Preference	Average			
	z^C/z^*	z^{GA}/z^*	z^{LPT}/z^*	z^{BAOG}/z^*
A	0.564	0.574	0.908	0.968
B	0.547	0.564	0.903	0.968
C	0.812	0.626	0.931	0.967
D	0.496	0.534	0.915	0.970

Product Conflicts

In any scenario, our algorithm outperforms other three algorithms. Table 5 shows all algorithms maintain their performance level regardless of change in conflict probability.

Table 5. The Effect of Conflict Probability

Conflict Probability	Average			
	z^C/z^*	z^{GA}/z^*	z^{LPT}/z^*	z^{BAOG}/z^*
0%	0.602	0.577	0.915	0.969
10%	0.606	0.577	0.914	0.968
25%	0.607	0.568	0.914	0.968

Overall Performance Comparison and Findings

In the experiment, BAOG performs the best when the number of time slots is between 1080 and 1440, and the number of commercials is 10. Unlike other three benchmark algorithms, BAOG remains its high-performance regardless of the number of passenger types, passenger preferences, and product conflicts. Table 6 shows that the average performance of each of four algorithms over linear relaxation solution under 324 scenarios. Despite cyclic broadcasting and the genetic algorithm perform slightly better than BAOG does in total effective exposure, BAOG performs best in the minimum effective exposure and demonstrates its improvement in both minimum effective exposure and total effective exposure compared to the greedy algorithm. Moreover, shown in Introduction, the average number of people commuting with buses in Taipei, Taiwan is 1,302,832 per day. Therefore, if BAOG replaces cyclic broadcasting and is applied to all the bus routes in Taipei, Taiwan, the advertiser who receives the minimum effective exposure will reach to more targeted customers by 472,928 people per day.

Finally, the average computation time of all algorithms are also shown in Table 6. BAOG takes about 4 minutes to complete one-time planning for a bus route. As normally there is at least an hour break between shifts of a bus, the 4-minute computation time that BAOG requires to schedule for a round bus trip is quite acceptable. Overall, BAOG maintains its high performance under any scenarios and demonstrates its capability of handling both efficacy and efficiency.

Table 6. Overall Performance

	Minimum Effective Exposure	Total Effective Exposure	Average Computation Time (secs)
Cyclic Broadcasting	0.605	1.144	0.02
Genetic Algorithm	0.574	1.111	323.76
Greedy Algorithm	0.914	0.985	1.17
BAOG	0.968	1.0002	253.38

Case Study

In our case study, we work with a media agency named OOHA,[2] founded in 2015. OOHA is a startup company which chooses to develop its own business in Taipei City and New Taipei City.

Every media agency has its own platform to display commercials, such as televisions or social networks. For OOHA, advertising on buses is a huge part of their business, and they collaborate with Capital Bus,[3] the largest bus company in Taipei, Taiwan. Nowadays, there are more and more digital billboards deployed on buses, shown in Figure 1. As of December 2018, OOHA deployed their billboards on more than 30 routes that fully cover northern Taiwan. For example, Line-920 bus goes from northwestern to southeastern New Taipei City. Figure 2 shows the route map of Line-920 bus. The average time of one-way bus trip is around 2 hours, during which OOHA displays commercials of various products, from medical to real estate commercials.

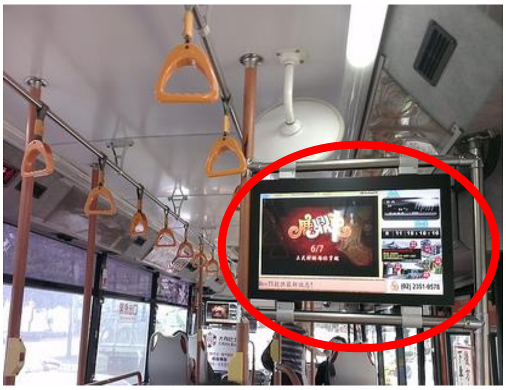

Figure 1. Digital Billboards on Buses (taken by the Authors)

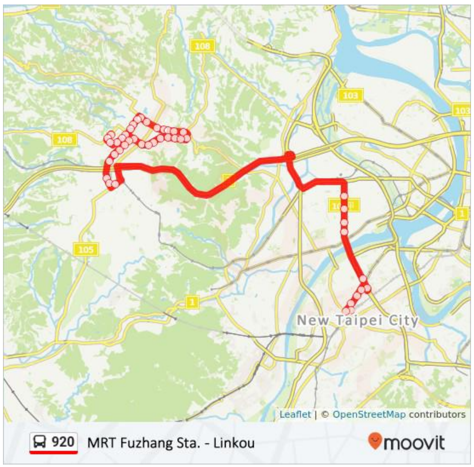

Figure 2. Line-920 Bus Route Map[4]

[2] For more information about OOHA, please see *http://oohaweb.com/*

[3] For more information about Capital Bus, please see *http://www.capital-bus.com.tw/*

[4] Screenshot from *https://moovitapp.com/index/en/public_transit-Taipei-3843*.

It is OOHA that sees the chance in advertising on transportation. They start working with bus companies, putting digital billboards on buses. Aside from the deployment of digital billboards, they are also in charge of software used to display commercials. Currently, they adopt cyclic broadcasting to schedule commercials. Without a doubt, this method is both straightforward and easy to implement; however, OOHA is thinking to improve the existing scheduling method because of the new technology they adopt.

Shown in Table 7 and Figure 3, OOHA recently starts to install camera on buses to collect passenger data, including demographic profiles, such as age and gender. With those additional data, they can know passenger composition and improve their scheduling method. Therefore, we use our algorithm to help them achieve this.

To dispel privacy concerns, the photos taken by the camera are directly sent to the facial recognition system operated by OOHA. After the system analyzes each passenger's demographic profile, these photos will then be completely deleted immediately to follow the law set by the government. The passenger data is processed without any human interventions. Our research team has never seen any of these files. We only receive and use the processed passenger data shown in Table 7.

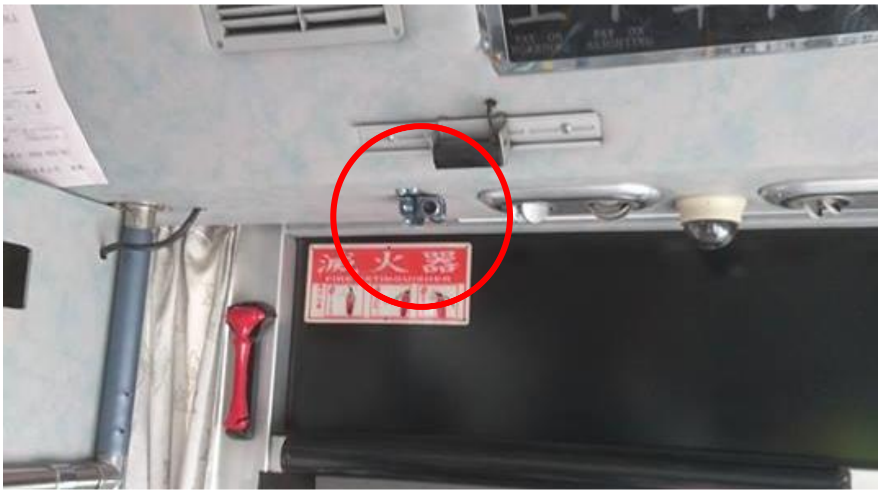

Figure 3. The Camera Installed on Buses (taken by the Authors)

Table 7. Passenger Data Example

Route	Longitude	Latitude	Gender	Age	Direction	Date	Time
920	121.3711	25.065	Female	22	Outbound	2018/03/29	9:05:29
920	121.3711	25.065	Female	28	Outbound	2018/03/29	9:05:30
920	121.3695	25.065	Female	22	Outbound	2018/03/29	9:07:43
920	121.3658	25.077	Male	39	Outbound	2018/03/29	12:39:44

Data Pre-processing and Prediction

Before prediction, we found that the camera may capture the same passenger multiple times in a few seconds, which suggests that these repetitive data should be filtered out to avoid affecting prediction result. Since the location is displayed by the latitude and longitude coordinates, we map the coordinates to the nearest bus stop. Last, we classify passengers by age and gender. In the end result of data pre-processing, the passenger data captured by the camera is transformed into the onboarding passenger composition at each bus stop.

In prediction, as Table 8 shows, we forecast the passenger composition between bus stops. It seems reasonable that we simply take the average composition of historical data as the prediction result. Yet, this method neglects the influence of date, weather, and rush hour on passenger composition. Hence, we take these attributes into consideration and adopt K-means to cluster historical data into groups.

Table 8. Passenger Composition Example

Passenger Type	1st to 2nd stop	2nd to 3rd stop	3rd to 4th stop	4th to 5th stop
20-30 Male	1	3	2	0
20-30 Female	0	1	2	4
30-40 Male	2	5	3	1
30-40 Female	3	4	9	4

When a bus is about to leave for the first stop, route, direction, date, current time, current weather and status (whether it is at rush hour) are required attributes for K-means to classify this bus into its group. We take the centroid of the group as the passenger composition between stops for the bus.

Optimization

We then apply passenger composition and passenger preference provided by OOHA to compute effective exposure each advertisement receives and total effective exposure generated by the schedule. Table 9 shows the example of passenger preference. BAOG schedules for the one-way trip before the bus leaves the first stop and provides nearly optimal schedule for OOHA to broadcast commercials within given time limit.

Table 9. Passenger Preference Example

Passenger Type	Real Estate Commercial	Bedding Commercial	Cosmetics Commercial	Financial Fund Commercial
20-30 Male	3	1	2	4
20-30 Female	2	3	5	3
30-40 Male	5	5	1	4
30-40 Female	4	5	5	2

As we take passenger composition into account and BAOG aims to maximize minimum effective exposure, the schedule helps advertisers reach their target customers and preserves fairness among advertisers. When the number of different commercials hikes, OOHA can apply BAOG to effectively meet their clients’ needs. Table 10 shows the result of applying all algorithms, including the benchmark ones, on a sample of 100 one-way Line-920 trips from 6 a.m. to 8 p.m. during May to June, 2018. It can be observed that BAOG indeed significantly outperforms all other benchmark algorithms.

Table 10. The Performance of 100 One-Way Trips of Route 920

	Minimum Effective Exposure	Total Effective Exposure
Cyclic Broadcasting	0.623	1.022
Genetic Algorithm	0.790	1.006
Greedy Algorithm	0.891	0.918
BAOG	0.989	0.996

Conclusion

In this study, we consider an advertisement scheduling problem on public transportation with fairness among advertisers as the main focus. Since we adopt effective exposure to evaluate allocation fairness, our problem is formulated as an integer program to maximize the minimum effective exposure subject to real-world constraints. For constraints, we consider excessive advertisement repetition and product conflicts. We develop our algorithm BAOG to solve the problem. We first design a heuristic based on the LPT rule to generate an initial solution and then develop a branch-and-bound algorithm by adjusting optimality gap over runtime to obtain the best possible schedule within the time limit. On average, BAOG performs better than traditional cyclic broadcasting, the genetic algorithm, and the greedy algorithm in numerical experiments. Moreover, we demonstrate that BAOG is more appropriate when the majority of passengers belong to a certain passenger type and the travel duration is long. Finally, we implement BAOG in collaboration with OOHA, a Taiwanese media agency. In sum, our algorithm BAOG demonstrates its possible applicability in practice and its capability of generating direct benefits for media agencies while handling both efficiency and efficacy.

Some investigations may further extend this study. For instance, to evaluate when to adjust the optimality gap, we can also adopt the number of visited nodes with nearly same objective or put it into combination. For other real-world problems, as some advertisers define their target market by locations, future works may also consider area limitations.

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