The Relation Between the Phase Transition in Ising Type Agent Model and Its Implication to the Growth Curve

Research-in-Progress

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Abstract

It has been said that many ICT related services such as SNS have network externality. It has also been said that the network externality has the chilling effect and a service cannot gain many users at the start-up stage, which means that the service providing firm must keep investing even if it does not have enough income at its start-up. This chilling effect can be explained by agent models. It is also known that agent model has the feature of phase transition. In this paper, we investigate the effect of the phase transition and its relation to the chilling effect. It is shown that there is no chilling effect under the process of phase transition while keeping other features of externality such as fixed shares. This implies that a service providing firm can obtain large number of users at the start-up stage with taking the advantage of the fixed share.

Keywords: Phase transition, network externality, chilling effect, Ising model, growth curve

Introduction

Network externality is defined to be the phenomenon that the utility of a service is highly dependent on the number of the users. It is often said that network externality plays an important role in many Web based services. A typical example is social networking service (SNS). In SNS, the service with large number of users can provide more utility than that with fewer users, because the users of the service can have connection with more other users. This leads to the winner-take-all phenomena (Frank and Cook 1995) and the firm with the largest share can have more users while the other firms with fewer users lose the shares.

At the same time, it is also often said that network externality has the "chilling effect" (Goldenberg et.al., 2010). A service cannot gain large number of users for some period after it starts the service. Network externality means that a service with no user has no utility. For example, telephone service makes no sense if there is only one user, because the user can call nobody. Due to this effect, the service providing firm cannot make profit at its start-up stage.

If the service can gain some number of users, its utility gradually increases. Then, the service can attract new other users if the new users know the users that have already joined the service. This effect gives the "S-shaped" growth curve. There have been many typical examples of this network externality: the diffusion of fax machines, player-to-player network gaming services, SNS etc. Figure 1 shows the growth of the SNS-like blog service in Japan. The growth can be split to three stages.

1. Start-up stage: the service has started but cannot gain enough users.
2. Growth stage: once the service gains some amount of users, the utility increases and attracts more users. Through this process, it gains more and more users and the service grows rapidly.

3. Saturation stage: the service (and other similar services provided by competing service providers) covers almost all of the potential users. The share tends to be fixed due to the effect of network externality.

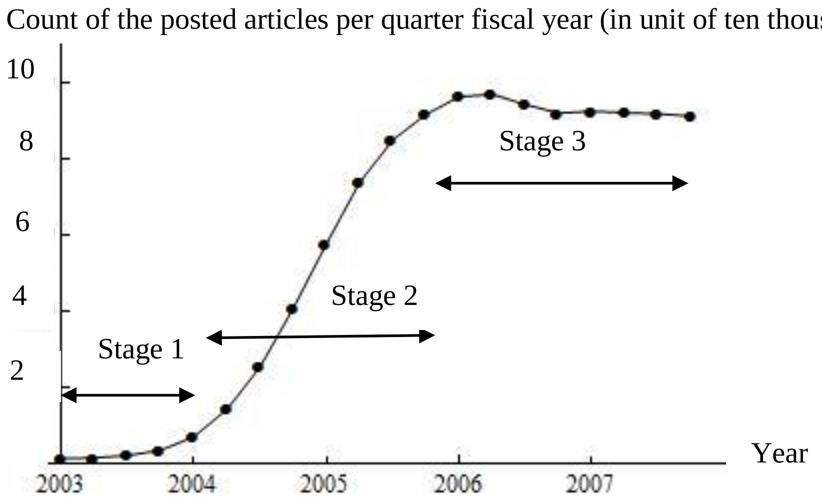

Figure 1. **The growth curve of SNS-like blog services in Japan. The growth can be split into three stages. (Quoted from the report by the Institute for Information and Communications Policy of Japan, 2009)**

This feature means that a service provider must endure the deficit and keep investing during the stage 1. Although it is possible that the firm goes bankrupt during this stage, it can obtain large number of users if it can go through this stage and move to the stage 2. Then, the firm can be a dominant player in the market and can have stable share at stage 3.

So far, the phenomena under network externality have been analysed from several aspects as is shown in the next section. Some of those works use agent simulations. In the agent simulations, many agents (that simulates customers and service providers) interact with each other and make decisions. By the accumulation of decisions of all the agents (such as joining a new service), macroscopic quantities such as the growth curve can be given.

One of the simplest kind of agent models is Ising type agent model, in which only the nearest neighbouring agents interact with each other. Ising model is known to have phase transition and shows qualitatively different macroscopic behaviours depending on the parameters.

So far, it has not been clarified how the phase transition affects the phenomena related to the network externality, both theoretically and empirically. In this paper, we analyse the effect of network externality within the framework of Ising model and focus on how the phase transition affects the behaviours of growth curve. Our research question is:

1. Whether or not the phase transition affects the growth curve.
2. If it affects, what kind of effect does it have to the start-up stage of a new service?

This paper is organized as following. In the next section, we show literature review, in which we show the works related to network externality and agent models. In the third section, we explain the details of Ising model that is used in this work. The fourth section is devoted to the results of numerical simulations, where we show the phase transition and its relation to the growth curve based on Ising model. In the last section, we show the summary and future prospects.

Literature Review

The study of network externality started analytically. Leibenstein (1950) analysed the effect of network externality as bandwagon effect. Bass (1969) discussed network externality from the viewpoint of commodity diffusion. Katz and Shapiro (1985) made discussion based on the two-period model. The chilling effect was first studied by Rohlfs (1974), in which he discussed the importance of lowering the initial barrier of introducing a new service at its start-up stage.

In the recent twenty years, numerical analysis based on agent models has made progress (Oomes 2003, Outkin 2003, Fang and Wang 2012). This is mainly due to the availability of fast computing resources. In these agent model analyses, each agent represents an entity to make a decision such as an individual to decide whether or not to purchase a new service. Each agent interacts with each other and makes a decision. The sum of these microscopic interactions gives the macroscopic quantities such as the growth curve and the shares. In general, the interaction between the agents has the effect to have the agents be in the same states: purchasing the same service. Due to this effect, many agents do not join a new service because many neighbouring agents have not joined the service yet (Goldenberg et. al., 2010).

Ising model is one of the simplest kind of agent models, in which the interaction is restricted to the ones between the nearest neighbouring agents. Since this model is based on the simple microscopic interaction between the decision makers that is described as agents, it can give more detailed time-dependence compared to the previous analytical approaches by Bass etc. shown above. Since the interaction in Ising model is simple, one can obtain analytical solutions if the environment is also simple (Onsager 1945, Kapusta 2011). It has been shown that the model shows second-order phase transition: there appears macroscopic segment in which many of the agents join the same service if the interaction between the agents is larger than a threshold. The segment disappears if the interaction is small even when it is not zero. In this situation, the agents have no correlation and join services randomly. This is brought about from the effect of the thermal noise introduced by the temperature parameter in the model. This parameter gives fluctuation into the model and reduces the effect to align the states of the agents.

Since the structure of Ising model is simple, it can be easily handled numerically and has been applied to many research areas of social science. Oh and Joen (2007) used this model to analyse the stability of open software development system. Zhou and Sornette (2007) studied the financial market based on Ising model. Zalkan et. al. (2009) and also Hokamp et. al. (2010) discussed the mechanism of tax evasion using this model. Laciana and Rovere (2011) studied the effect of new technology by this model. In these previous works, however, the relation of these phenomena and the phase transition is not shown.

In the next section, we explain the details of the Ising model and also explain how to carry out numerical simulation based on this model.

Ising Model and the Procedure of Simulations

In this section we show the structure of Ising model. As is shown in the previous sections, it is widely used in many areas, and there is a textbook for this and related models (Kapusta, 2011). In this section, we explain Ising model in the framework of investigating the economic phenomena.

Ising model was first invented to study the ferromagnetism. An iron bar, for example, becomes a permanent magnet, but loses its magnetism if the temperature is high. Qualitatively, this phenomenon can be interpreted as following.

Each iron atom has small magnetism. If the magnetism of all the atoms is aligned in the same direction, the iron bar becomes a permanent magnet (Figure 2). If the temperature is high, the thermal fluctuation breaks the alignment of the small magnets. As is shown in Figure 3, the direction of the magnetism of the iron atoms becomes random and the iron bar loses macroscopic magnetism.

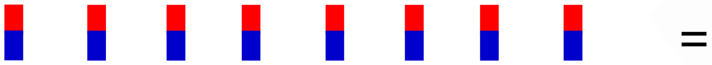


Figure 2. If the small magnetism of iron atoms is aligned, the iron bar becomes a permanent atom.

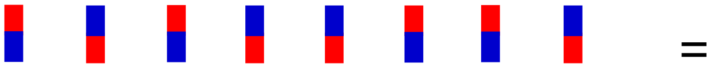


Figure 3. If the directions of the magnetism of the atoms are random, the total magnetism is lost.

Once the macroscopic magnetism is lost, it does not come back even if the temperature is lowered. The magnetism is frozen at random microscopic states.

In order to consider this magnetism, Ising model was invented. In the original Ising model of Physics, two kinds of interactions are introduced: the interaction between the agents and the interaction with external magnetic field. The state of each atom is denoted by S_i. S_i=+1 means that the magnetism of the atom specified by the index *i* is upward (north pole upward as is shown in Figure 2). S_i=-1 means that the north pole is downward. Hereafter, the state S_i=+1 is denoted by an upward arrow and S_i=-1 by a downward arrow (Figure 4).

Figure 4. The S_i=+1 state is denoted by the left upward arrow. It is identical to the small magnet in Figure 2. The S_i=-1 state is denoted by the right downward arrow.

The utility of the system (iron bar in the case above) is given by

$$U=U_I+U_H \qquad (1),$$

where U_I and U_H is given by

$$U_I = \sum_i \{J \cdot S_{i-1} \cdot S_i\} \qquad (2)$$

and

$$U_H = \sum_i H \cdot S_i \qquad (3).$$

The parameters *J* and *H* give the strength of the terms. Both *H* and *J* are positive numbers. U_I gives *J* for $S_i=S_{i-1}$. This means that the agents favour to be in the same state as the neighbouring agents. In other word, U_I gives the effect of network externality, while U_H gives the utility that is irrelevant to the states of the neighbouring agents.

For example, agents prefer being in S_i=+1 if the neighbouring agents are in S_{i-1}=+1. This gives the effect that all the directions of the magnetism of the iron atoms are aligned.

In Ising model, the dynamic behaviours of the agents are given stochastically. The probability that the state with a specific *U* is given by

$$\exp(U/T) \qquad (4),$$

where T is the temperature parameter that shows the strength of thermal fluctuation. Large T means that there is a big noise that makes the directions of the magnetism of the iron atoms being random.

So as to apply this model to analysing economic phenomena, S_i=-1 is interpreted that the agent specified by i is not purchasing a service, S_i=+1 purchasing it. The process that a service grows is the transition from the state that all the agents are in -1 state (Figure 5) to the state that many of the agents are in +1 state (Figure 6).

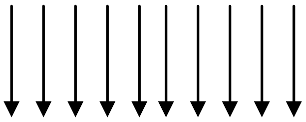

Figure 5. The initial state that no one is purchasing a service.

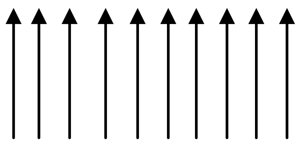

Figure 6. The final state that many of the agents are purchasing the service.

If H and J are large enough, the utility given by the equation (1) is maximized when all the agents are in +1 shown in Figure 6. Due to the effect of U_I, however, the initial state shown in Figure 5 does not go to the state in Figure 6 swiftly. The agents prefer staying at -1 state because the neighbouring agents are all in -1 state at the initial state. This is the chilling effect. The effect of U_H drives the system away from the initial state. If the final state is established, however, it is stable even if H is set to zero. This is the situation that an iron bar keeps being a magnet if a strong external magnetic field is given and then is removed. Thus the parameter H is related to the speed to go to the final state, but is irrelevant to the final state.

At the same time, it is known that Ising model shows phase transition. If J is large, the final states are those shown in Figure 6, in which all of the agents are in +1 state. If J is small (but not zero), however, it is known that the final state is such as shown in Figure 7, i.e. random state. The total (macroscopic) magnetism is zero (in other words, the percentage of the users purchasing a service is 50%).

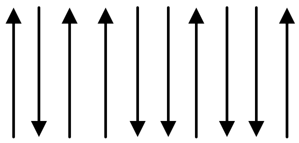

Figure 7. The final state for small *J* in which the alignments of the magnetism of the atoms are random.

The way that the initial state moves to the final state is given by numerical simulation. It is carried out in the following procedure.

1. All the S_i's are set to -1 (initial state).
2. The probability that an agent with index i takes the state +1 is given by the equation (4). For example, U_H is $+H$ for S_i=+1 and $-H$ for S_i=-1, and U_I gives $-J$ for S_i=+1 (because the neighbouring S_{i-1} is -1) and $+J$ for S_i=-1. If H=J=1, U=H-J=1-1=0 for S_i=+1 and -1+1=0 for S_i=-1. Equation (4) is exp(0) both for +1 and -1 state. This means the probability for S_i=+1 state is 50%.
3. Create a random number r ($0<r<1$) and compare it with the probability given in the procedure 2. In the example above, if $r>0.5$ S_i=+1 is chosen.
4. The procedure 3 is repeated for all the agents and the new set of S_i's is given.
5. The procedure form 2 to 4 is repeated. The count of repetition is interpreted to be the time elapsed from the beginning of the simulation. It is repeated until the states do not change and the stable state in established.

In this section, one-dimensional Ising model was shown for simplicity. In the real simulations of the previous works, however, two-dimensional Ising model was used. Also in the simulation shown in the next section, we use two-dimensional model, in which the agents are allocated in lattice.

In the next section, we show the results of the numerical simulations and its interpretation.

Results of Numerical Simulations and Its Interpretation

As is shown in the previous sections, we focus on the relation between the phase transition and the growth curve. In the calculation below, we redefine the parameters H/T and J/T as H and J, because H and J always appear in these expressions within the Ising model.

At first, we carry out simulation for various parameters and confirm that there exists phase transition. The result is shown in Figure 8. In this calculation, the parameter H is set to 0 and is irrelevant to the results. It is shown that the effect of the interaction between the agents is strong enough to keep all the agents to +1 states for $J>1$. On the other hand, the rate of the joining agents is 0.5 for $J<0.83$, which means that 50% of the agents are in +1 states and the rest 50% in -1 state. In other words, agents are not correlated and are in random states. The transition from the ordered state to random state takes place through the change of J from 1 to 0.83. Our aim is to know the growth curve for $0.83 < J < 1$.

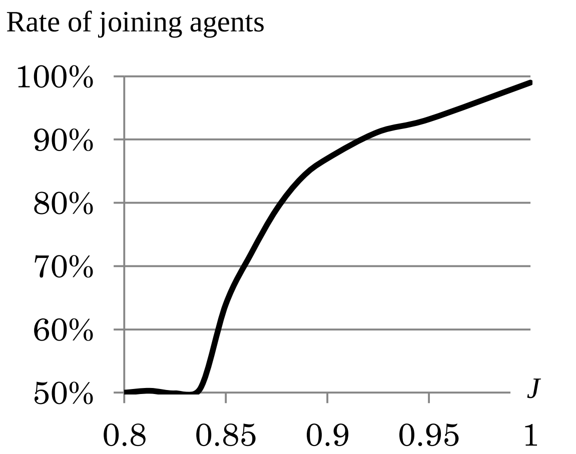

Figure 8. The phase transition in this model. It shows that almost all the agents join the service for $J>1$, whereas all the agents are in random states for $J<0.83$. The range $0.83<J<1$ is in the process of phase transition.

A typical growth curve is shown in Figure 9 for the parameter $J=2.0$. Since the effect of network externality is strong for this parameter, the growth curve is S-shaped. One can see that there is a chilling effect at the start-up stage, i.e. slow growth for the period less than 20 in the horizontal axis.

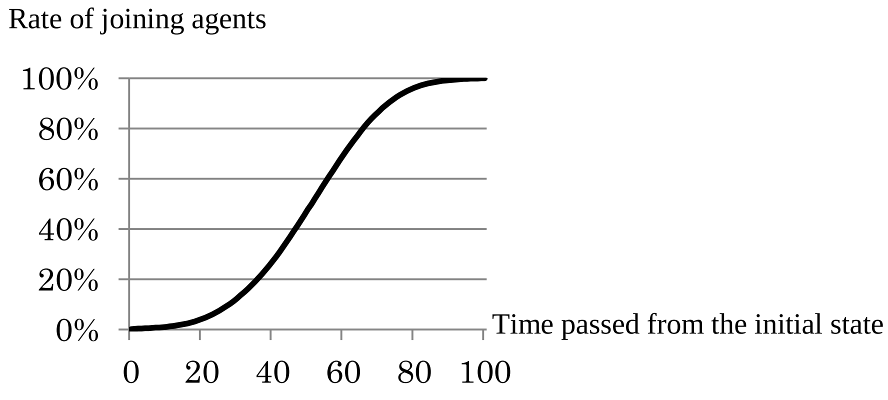

Figure 9. Typical S-shaped curve for *J*=2.0.

An example of growth curve during the phase transition is shown in Figure 10 for *J*=0.9. One can see that there is no chilling effect and the growth curve is almost linear at the start-up stage (stage 1). This implies that a service provider can gain some amount of users and the income for the investment even at the start-up stage. On the other hand, one can see that there is an effect of network externality at the saturation stage (i.e. the rate goes to 100% not 50%) and that the service provider can have large share at the saturation stage. This situation is favourable for small service providers, because large investment under deficit is not required but still the service providers can have fixed shares at saturation.

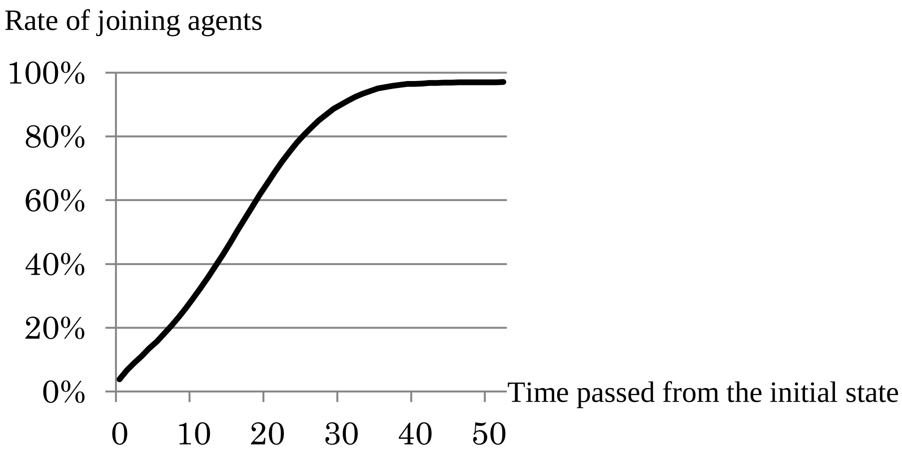

Figure 10. The growth curve for *J*=0.9. One can see that there is no chilling effect.

We have made simulation for other parameters and found that there are essentially the same trends for the parameters *J*>0.87.

Conclusion and Future Prospect

We investigated the growth curve based on Ising type agent model and found that there is no chilling effect for the parameters during the phase transition. This parameter is favourable for small venture firms because large amount of fund is not required to endure the initial investment under deficit. It is also favourable in the sense that there exists some effect of network externality at the saturation stage, and the firm that started the service earlier than other firms can have stably large share in the market.

All of these findings are made based on theoretical analysis. It is not known what kind of market fits the parameter shown in this work. To compare the theoretical result with the real world is one of the most important agenda to be done in the future. Also it is important to see if the same result can be obtained for other kind of agent models such as scale-free network etc.

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